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RESEARCH MEMORANDUM

ESTIMATION OF RANGE OF STABILITY DERIVATIVES FOR

CURRENT AND FUTURE PILOTLESS AIRCRAFT

By

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SUMMARY

Because of the interest in the use of mechanical-electrical devices for computing the stability characteristics of pilotless aircraft and guided missiles, an analysis was made of the probable range of the stability parameters of present and future missiles.

Included is a short review of the dynamic theory and some of the factors and assumptions influencing the stability derivatives and equations of motion.

In perusing this paper, one must remember that the derivatives are referred to the wing dimensions for conversion from forces and moments. Because some missiles may have very small wings compared to their bodies, the relative magnitudes of the derivatives (for a specific aircraft) with respect to each other are of more significance than their absolute magnitudes.

INTRODUCTION

The mathematical solution of the stability of an automatically guided and stabilized missile involves a number of assumptions concerning the automatic pilot and is tedious and complex and requires mathematical skill not ordinarily available. Interest has, therefore, centered upon the use of "analogue" devices which duplicate the stability of the missile under investigation by mechanical, electrical, electronic, or other physical means. These devices are usually so arranged as to obtain knowledge of a missile's motions following a disturbance by direct measurement.

One such device is called a "flight table" and is a mechanical and electrical arrangement which includes the automatic pilot of the missile under investigation as part of its mechanism. In order to duplicate properly the stability characteristics of the missile with

such a device, it is also necessary to provide for duplication of the aerodynamic and mass characteristics of the missile. The remainder of the text presents the results of a survey made to estimate the probable range of the stability parameters of existing and future missiles in a form suitable for the design of a flight table.

SYMBOLS

m mass, slugs (W/g)

W weight, pounds

g acceleration of gravity, feet per second per second (32.2)

v sidewise velocity, feet per second (positive to the left)

V forward velocity, feet per second

S wing area, square feet

b wing span, feet

t time, seconds

l tail length, feet (distance from center of gravity to center of pressure of tail)

c mean aerodynamic chord, feet

u incremental velocity, feet per second (ΔV)

$u' = \dfrac{u}{V}$

q_o dynamic pressure, pounds per square foot $\left(\dfrac{1}{2}\rho V^2\right)$

L lift force, pounds

D drag force, pounds

M pitching moment, foot-pounds (positive nose up)

C_L lift coefficient $\left(L/q_o S\right)$ and $\left(W \cos \gamma \cos \phi / q_o S\right)$

C_D drag coefficient $\left(D/q_oS\right)$

$T_c{}'$ effective thrust coefficient $\left(\dfrac{\text{Effective thrust}}{q_oS}\right)$

C_m pitching-moment coefficient $\left(M/q_oSc\right)$

Z   normal force, pounds (positive downward)

X   longitudinal force, pounds (positive forward)

C_Z normal-force coefficient $\left(Z/q_oS\right)$

C_X longitudinal-force coefficient $\left(X/q_oS\right)$

C_Y lateral-force coefficient $\left(\dfrac{\text{Lateral force}}{q_oS}\right)$

C_l rolling-moment coefficient $\left(\dfrac{\text{Rolling moment}}{q_oSb}\right)$

C_n yawing-moment coefficient $\left(\dfrac{\text{Yawing moment}}{q_oSb}\right)$

ρ mass density of air, slugs per cubic foot

α angle of attack of aircraft reference line to flight path

θ inclination of aircraft reference axis from horizontal plane (fig. 1)

β angle of sideslip $\left(\dfrac{v}{V}\right)$

ψ angle of yaw

ϕ angle of bank

q pitching angular velocity $\left(\dfrac{d\theta}{dt}\right)$

r yawing angular velocity $\left(\dfrac{d\psi}{dt}\right)$

p   rolling angular velocity $\left(\dfrac{d\phi}{dt}\right)$

ϵ inclination of longitudinal principal axis from aircraft reference line (fig. 1)

γ inclination of flight path from horizontal

η inclination of longitudinal principal axis from longitudinal stability axis $(\eta = \theta - \gamma - \epsilon)$

δ control surface deflection, positive when trailing edge moves downward or to left

k_X radius of gyration about principal longitudinal axis, feet

k_Z radius of gyration about principal normal axis, feet

k_Y radius of gyration about Y-axis, feet

$$K_X = \frac{k_X}{b}, \quad K_Z = \frac{k_Z}{b}, \quad K_Y = \frac{k_Y}{c}$$

μ relative density of airplane $(m/\rho Sb)$

μ_c relative density of airplane $(m/\rho Sc)$

s   nondimensional time parameter $\left(\dfrac{tV}{b}\right)$

s_c nondimensional time parameter $\left(\dfrac{tV}{c}\right)$

D   differential operator $\left(\dfrac{d}{ds}\right)$

D_c differential operator $\left(\dfrac{d}{ds_c}\right)$

$$C_{m_{u'}} = \frac{\partial C_m}{\partial u'}, \quad C_{Z_{u'}} = \frac{\partial C_Z}{\partial u'}, \quad C_{X_{u'}} = \frac{\partial C_X}{\partial u'}$$

$$C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha}, \quad C_{Z_\alpha} = \frac{\partial C_Z}{\partial \alpha}, \quad C_{X_\alpha} = \frac{\partial C_X}{\partial \alpha}$$

$$C_{m_\delta} = \frac{\partial C_m}{\partial \delta}, \quad C_{Z_\delta} = \frac{\partial C_Z}{\partial \delta}, \quad C_{X_\delta} = \frac{\partial C_X}{\partial \delta}$$

$$C_{m_\theta} = \frac{\partial C_m}{\partial \theta}, \quad C_{Z_\theta} = \frac{\partial C_Z}{\partial \theta}, \quad C_{X_\theta} = \frac{\partial C_X}{\partial \theta}$$

$$C_{m_{D\alpha}} = \frac{\partial C_m}{\partial \frac{\dot{\alpha}c}{2V}}, \quad C_{Z_{D\alpha}} = \frac{\partial C_Z}{\partial \frac{\dot{\alpha}c}{2V}}, \quad C_{X_{D\alpha}} = \frac{\partial C_X}{\partial \frac{\dot{\alpha}c}{2V}}, \quad \dot{\alpha} = \frac{d\alpha}{dt}$$

$$C_{m_{D\delta}} = \frac{\partial C_m}{\partial \frac{\dot{\delta}c}{2V}}, \quad C_{Z_{D\delta}} = \frac{\partial C_Z}{\partial \frac{\dot{\delta}c}{2V}}, \quad C_{X_{D\delta}} = \frac{\partial C_X}{\partial \frac{\dot{\delta}c}{2V}}, \quad \dot{\delta} = \frac{d\delta}{dt}$$

$$C_{m_q} = \frac{\partial C_m}{\partial \frac{qc}{2V}}, \quad C_{Z_q} = \frac{\partial C_Z}{\partial \frac{qc}{2V}}, \quad C_{X_q} = \frac{\partial C_X}{\partial \frac{qc}{2V}}, \quad q = \frac{d\theta}{dt}$$

$$C_{Y_\beta} = \frac{\partial C_Y}{\partial \beta}, \quad C_{n_\beta} = \frac{\partial C_n}{\partial \beta}, \quad C_{l_\beta} = \frac{\partial C_l}{\partial \beta}$$

$$C_{Y_\delta} = \frac{\partial C_Y}{\partial \delta}, \quad C_{n_\delta} = \frac{\partial C_n}{\partial \delta}, \quad C_{l_\delta} = \frac{\partial C_l}{\partial \delta}$$

$$C_{Y_{D\delta}} = \frac{\partial C_Y}{\partial \frac{\delta b}{2V}}, \quad C_{n_{D\delta}} = \frac{\partial C_n}{\partial \frac{\delta b}{2V}}, \quad C_{l_{D\delta}} = \frac{\partial C_l}{\partial \frac{\delta b}{2V}}, \quad \dot{\delta} = \frac{d\delta}{dt}$$

$$C_{Y_r} = \frac{\partial C_Y}{\partial \frac{rb}{2V}}, \quad C_{n_r} = \frac{\partial C_n}{\partial \frac{rb}{2V}}, \quad C_{l_r} = \frac{\partial C_l}{\partial \frac{rb}{2V}}, \quad r = \frac{d\psi}{dt}$$

$$C_{Y_p} = \frac{\partial C_Y}{\partial \frac{pb}{2V}}, \quad C_{n_p} = \frac{\partial C_n}{\partial \frac{pb}{2V}}, \quad C_{l_p} = \frac{\partial C_l}{\partial \frac{pb}{2V}}, \quad p = \frac{d\phi}{dt}$$

Subscripts:

a   aileron

r   rudder

w   wing

THEORY

The dynamics of mechanical flight are assumed amenable to analysis on the basis of the theory of small oscillations. On this assumption then, one equation is written for each degree of freedom of the motion. Simultaneous solution of the equations permits calculation of the motion of the airplane after a disturbance. The terms defining the changes in aerodynamic forces and moments due to a deviation are referred to as "stability derivatives" and are partial derivatives.

Assumptions.- In the application of the theory the following assumptions are made:

(1) The combined aerodynamic effect of two or more components of motion is assumed equal to the algebraic sum of the separate effects of the individual components.

(2) The changes in aerodynamic forces and moments due to a deviation are assumed directly proportional to the deviation (that is, the stability derivatives are constants).

(3) Secondary effects such as those involving the products of two or more second-order quantities are neglected.

(4) The values of the aerodynamic factors are assumed to be unaffected by the linear and angular accelerations.

(5) The lateral motion is assumed to be independent from the longitudinal motion.

Equations of Lateral Motion

The nondimensional equations for lateral motion can be written for the azimuth oriented flight-path axes (stability axes) shown in figure 2.

These take the form of

(a) $\sum C_l$

$$C_{l\delta_r} \delta_r + \left(C_{l\delta_\alpha} + C_{lD\delta_\alpha} D\right)\delta_\alpha \div C_{l\beta}\beta + \frac{1}{2}C_{l_p} D\phi + \frac{1}{2}C_{l_r} D\psi$$

$$= 2\mu\left[\left(K_X^2 \cos^2\eta + K_Z^2 \sin^2\eta\right)D^2\phi + \left(K_Z^2 - K_X^2\right)\cos\eta \sin\eta D^2\psi\right] \quad (1)$$

(b) $\sum C_n$

$$\left(C_{n\delta_r} + \frac{1}{2}C_{nD\delta_r} D\right)\delta_r \div C_{n\delta_\alpha}\delta_\alpha + C_{n\beta}\beta + \frac{1}{2}C_{n_p} D\phi + \frac{1}{2}C_{n_r} D\psi$$

$$= 2\mu\left[\left(K_Z^2 \cos^2\eta \div K_X^2 \sin^2\eta\right)D^2\psi \div \left(K_Z^2 - K_X^2\right)\cos\eta \sin\eta D^2\phi\right] \quad (2)$$

(c) $\sum C_Y$

$$\left(C_{Y\delta_r} + \frac{1}{2}C_{YD\delta_r} D\right)\delta_r + C_{Y\delta_\alpha}\delta_\alpha \div C_{Y\beta}\beta + \frac{1}{2}C_{Y_p} D\phi \div \frac{1}{2}C_{Y_r} D\psi + C_L\phi$$

$$+ C_L\psi \tan\gamma = 2\mu\left(D\beta + D\psi\right) \quad (3)$$

where δ, the control movement caused by the gyro, is usually a function of ϕ and ψ. Simultaneous solution of the differential equations (1), (2), and (3), will define the lateral motion of an airplane after disturbances in terms of the variables ϕ, ψ, and β provided that the stability derivatives $C_{n\beta}$, C_{l_p}, and so forth and

the mass and dimensional characteristics of the missile under investigation are known.

Equations of Longitudinal Motion

The equations of motion of a flying body about its longitudinal flight axes may be written in the form

(a) $\sum c_X$

$$\left(C_{X_\delta} + \frac{1}{2}C_{X_{D_\delta}} D_c\right) \delta + C_{X_u}u' + \left(C_{X_\alpha} + \frac{1}{2}C_{X_{D_\alpha}} D_c\right)\alpha$$

$$+ \left(C_{X_\theta} + \frac{1}{2}C_{X_q} D_c\right)\theta = 2u_c D_c u' \tag{4}$$

(b) $\sum c_Z$

$$\left(C_{Z_\delta} + \frac{1}{2}C_{Z_{D_\delta}} D_c\right) \delta + C_{Z_u}u' + \left(C_{Z_\alpha} + \frac{1}{2}C_{Z_{D_\alpha}} D_c\right)\alpha$$

$$+ \left(C_{Z_\theta} + \frac{1}{2}C_{Z_q} D_c\right)\theta = 2\mu_c D_c (\alpha - \theta) \tag{5}$$

(c) $\sum c_m$

$$\left(C_{m_\delta} + \frac{1}{2}C_{m_{D_\delta}} D_c\right) \delta + C_{m_u}u' + \left(C_{m_\alpha} + \frac{1}{2}C_{m_{D_\alpha}} D_c\right)\alpha$$

$$+ \left(C_{m_\theta} + \frac{1}{2}C_{m_q} D_c\right)\theta = 2\mu_c K_Y^2 D_c^2 \theta \tag{6}$$

As in the case of the lateral motions δ, the control deflection induced by the automatic system, is some function of the missile variables u', α, and θ. Note that $u' = \frac{\Delta V}{V}$.

ESTIMATION OF RANGE OF STABILITY DERIVATIVES

Lateral-Stability Derivatives

A summary of the lateral-stability derivatives for several specific subsonic missile designs is given as table I. All angles herein discussed are based on radian measure.

Side-force derivatives (equation (3)).- (a) C_{Y_β}: Values of C_{Y_β}, measure of the lateral force created by sideslip, for typical missile designs are given in table I. These data show that missiles designed along proportions of pursuit-type airplanes have C_{Y_β} values of the order of -0.4. In general, however, missiles have relatively large fuselages with respect to their wings and hence have greater side area to resist sideslipping and also a smaller wing area on which to base the coefficient. Missile no. 7 of table I $\left(C_{Y_\beta} = -5.15\right)$ and missile no. 3 of table I $\left(C_{Y_\beta} = -4.58\right)$ illustrate modern trends in this direction. In order to allow for future designs, therefore, it is recommended that provision be made to allow for a C_{Y_β} range of

$$-10.0 < C_{Y_\beta} < 0$$

(b) $C_{Y_{D_\delta}}$, C_{Y_p}, and C_{Y_r}: These derivatives are usually very small relative to the other lateral derivatives and are generally disregarded.

Side force due to automatic control.- (c) C_{Y_δ}: The side forces created by automatic control can be represented by $\delta \dfrac{\partial C_Y}{\partial \delta}$ or δC_{Y_δ} where δ can be a function of ϕ or ψ or both. No provision need be made to simulate δ since this is automatically taken care of by mounting the autocontrol on the flight table in the proper manner. However, it is necessary to duplicate C_{Y_δ}. This parameter depends on the type and size of control surface or unit used on the missile. Generally speaking, aileron deflection causes no side-force change. Application of rudder control will, however, introduce side forces. At least two guided missiles have utilized an autocontrol providing side-force changes with control deflection. The values for these two missiles are

$$C_{Y_\delta} = 0.516 \quad \text{Missile no. 3, table I}$$

$$C_{Y_\delta} = 1.60 \quad \text{Missile no. 7, table I}$$

The larger value represents the side force created by deflection of an all-movable tail surface, the other the side force created by a typical trailing-edge control. In gauging the range that this derivative can cover, it appears that the maximum will be reached by an all-movable wing. The side-force derivative will therefore be equal to the lift-curve slope of the wing. The maximum value of lift-curve slope is obtained on wings of infinite aspect ratio and is theoretically equal to 2π per radian. Near Mach number = 1.0, the lift-curve slopes of lower aspect ratio wings increase considerably, but probably none would have a lift-curve slope greater than 6.0. Consequently, the range of C_{Y_δ} covered by the flight table should be

$$-6.0 < C_{Y_\delta} < 6.0$$

It appears that this range will also cover the side forces created by a swiveling jet nozzle, or deflected jet. The side force developed by a nozzle delivering a thrust T and deflected 1 radian is approximately equal to the thrust which is the same order as the drag at top speed. In coefficient form then $C_{Y_\delta} = T_C' = C_D$, approximately.

On missiles designed for high speed the drag coefficient should not exceed 0.1 and hence the range of C_{Y_δ} previously noted should readily encompass the swiveling-nozzle-type control.

<u>Yawing-moment derivatives (equation (2))</u>.- (a) C_{n_β}: This parameter determines the weathercock stability of the missile. The values presented in table I show that fighter-type missiles have a value of C_{n_β} of about 0.075. Missiles having larger fuselage lengths relative to their wings have larger values of C_{n_β} ranging up to 0.5. Present trends in missile design indicate that the wing aspect ratio of some missiles is likely to decrease in the future. The aspect ratio A is equal to span^2/wing area and can be reduced by a decrease in span or an increase in wing area. A decrease in span is more probable than an increase in area inasmuch as wing areas are likely to be decreased to permit higher speeds at low altitudes. Because C_{n_β} is dependent on wing dimensions for its magnitude $C_{n_\beta} = \dfrac{\partial(N/qSb)}{\partial\beta}$, it is not unlikely

that C_{n_β} for designs of the future will be from two to four times
as large as present-day values. The recommended range of C_{n_β} is
therefore

$$-2.0 < C_{n_\beta} < 2.0$$

(b) C_{n_r}: Values of this derivative are shown in table I. These
data show that the C_{n_r} of a fighter-type missile is about -0.2 but
that the average is considerably larger. Values of C_{n_r} equal
to -0.66 and -0.55 were obtained for the large fuselage designs
(missile no. 3 and missile no. 5 of table I). It is possible to
estimate the probable range of C_{n_r} from the previously estimated
range of C_{n_β} as follows:

The contribution of the wings and fuselage to C_{n_r} is generally
negligible and the tail contributions can be expressed as

$$C_{n_r} = -2\frac{l}{b}\left(\Delta C_{n_\beta}\right)_{Tail}$$

Because $\left(\Delta C_{n_\beta}\right)_{Tail}$ is generally about $\frac{3}{2}C_{n_\beta}$ and assuming a tail
length equal to two times the wing span,

$$C_{n_r} = -6C_{n_\beta}$$

The range of C_{n_r} would therefore be of the order of 12.0. It is
conceivable that some missiles will be equipped with fore as well
as aft fins. In such cases C_{n_β} will not be so large. The
recommended range is therefore

$$-12.0 < C_{n_r} < 0$$

It should be pointed out that positive values of C_{n_r} cannot be
obtained aerodynamically below the stall.

(c) C_{n_p}: Values of C_{n_p} are presented in table I for present-
day designs. These data show that C_{n_p} is quite small with respect

to the other derivatives previously discussed. Even when a missile
is operating at high lift coefficient in a turn, it appears unlikely
that values of C_{n_p} greater than -0.4 could be reached. It is
possible that in some unusual cases positive values of C_{n_p} could
be obtained. This could happen on highly sweptback wings or at high
lift coefficients if asymmetric stalling takes place. Recommended
range for C_{n_p}:

$$-0.4 < C_{n_p} < 0.4$$

(d) C_{n_δ}: Yawing moments can arise through use of either the
aileron or rudder control. The aileron moments, however, are small
compared to those created by rudder. Typical values of $C_{n_{\delta_r}}$ for
modern missiles are of the order of -0.1. The previously estimated
range of C_{Y_δ} may be used to estimate the range $C_{n_{\delta_r}}$ from the
relationship $C_{n_{\delta_r}} = \frac{-l}{b} C_{Y_\delta}$. Assuming the maximum tail length to be
equal to two times the wing span, $C_{n_{\delta_r}}$ has a range of values twice
as large as C_{Y_δ}. Hence, it is recommended that

$$-12.0 < C_{n_{\delta_r}} < 12.0$$

Rolling-moment derivatives (equation(1).- (a) C_{l_β}: This deriva-
tive is the aerodynamic equivalent of wing dihedral. The values
of C_{l_β} shown in table I show that C_{l_β} ranges up to -0.143 for
modern designs. However, it is entirely probable that higher values
than -0.14 may be obtained in the future, particularly with highly
sweptback wings on high altitude, long range missiles operating at
fairly high lift coefficients. Therefore, the recommended range
of C_{l_β} is

$$-0.5 < C_{l_\beta} < 0.5$$

(b) C_{l_p}: Values of C_{l_p} are shown in table I. This informa-
tion shows that C_{l_p} is of the order -0.4. C_{l_p} is almost always
negative below the stall and is primarily determined by the aspect

ratio of the wing. Since the present trend is to decrease aspect ratio, C_{l_p} should also decrease; hence, the following range should cover future designs:

$$-0.7 < C_{l_p} < 0.2$$

(c) C_{l_r}: Values of C_{l_r} are shown in table I. These data show that C_{l_r} ranges up to about 0.2. C_{l_r} is always positive below the stall and is approximately equal to $C_L/3$. It appears that a satisfactory range of C_{l_r} is

$$-0.1 < C_{l_r} < 0.8$$

(d) C_{l_δ}: The aileron control is the chief source of rolling moments on conventional designs. Typical values for the rolling moments created by such controls on the aircraft in table I are of the order of -0.14. In general, automatically controlled missiles do not require a large amount of aileron control for stabilization or maneuverability. However, future designs may require large amounts of lateral control. In order to be conservative, therefore, provision should be made to cover a range of

$$-1.5 < C_{l_{\delta_a}} < 1.5$$

This estimate includes the extreme case of two wings of infinite aspect ratio deflected differentially to produce lateral control.

Miscellaneous information.- (a) K_X: The radius of gyration (in span lengths) about the X-axis will never exceed 1.0. Therefore,

$$0 < K_X < 1.0$$

K_Z: This factor is moderately small for missiles compared to airplanes owing to the fact that most of the missile mass is concentrated near its center of gravity. However, this may not be true in the future as present trends are to install heavy power plants in the rear of the missile. The recommended range is therefore

$$0 < K_Z < 5.0$$

(b) μ, relative density parameter: A convenient nondimensional mass parameter sometimes employed in theoretical calculations is the relative density ratio $\mu = \dfrac{m}{\rho S b}$. As missile weights and altitudes are increased, and wing areas and spans lessened, this parameter increases.

A normal value of μ for a conventional airplane is about 10.0. For missiles, the average is about 100 and has been as large as 200. It appears that μ values as large as 500 are not unlikely in the future. At high values of μ, above 500, large increases in this parameter cause negligible changes in the conditions for neutral stability in the oscillatory mode.

Longitudinal Derivatives

Longitudinal force derivatives (equation(4)).- (a) C_{X_u}: This derivative is a function of drag coefficient and thrust coefficient and will be of a magnitude of about twice these parameters. A typical value is about -0.056 for the JB-2 in level flight. In order to provide a safe margin for future designs the recommended range of C_{X_u}, is

$$-1.0 < C_{X_u}, < 1.0$$

(b) C_{X_α}: This derivative is approximately equal to $-C_{D_\alpha}$. Missiles may fly for short periods of time at fairly high values of C_L, and hence rather high values of C_{D_α} may be obtained. It therefore seems desirable to cover a range of C_{X_α} of

$$-2.0 < C_{X_\alpha} < 1.0$$

(c) C_{X_θ}: This derivative reaches a maximum in horizontal flight when it is equal to C_{D_α} but opposite in sign. It will always be smaller than C_{X_α}.

Recommended range for C_{X_θ} is

$$-1.0 < C_{X_\theta} < 0.5$$

(d) C_{X_δ}: In general, the longitudinal force change caused by a control deflection is negligible compared to the longitudinal forces created by airplane motion. However, if the wings themselves are used for control this factor could be of the order of C_{D_α}. Hence, recommended range for C_{X_δ} is

$$-2.0 < C_{X_\delta} < 2.0$$

(e) $C_{X_{D_\alpha}}$, $C_{X_{D_\delta}}$, C_{X_q}: These derivatives are usually small in comparison to the others and are generally neglected. However, it is possible that they may become large enough to be considered and are included in the equations for completeness.

Normal-force derivatives (equation (5)).- (a) $C_{Z_{u'}}$: This derivative is approximately equal to $-2C_L$. Hence, the recommended range for $C_{Z_{u'}}$ is

$$-2.4 < C_{Z_{u'}} < 2.4$$

(b) C_{Z_α}: C_{Z_α} is approximately equal to $-C_{L_\alpha}$. C_{L_α} is a maximum at 2π for wings. However, as in the case of C_{Y_β}, the fuselage lift should be considered. Hence, the recommended range for C_{Z_α} is

$$-10 < C_{Z_\alpha} < 0$$

(c) C_{Z_θ}: This derivative is a maximum in level flight when it is approximately equal to C_{L_α} but opposite in sign, hence its range is

$$-10.0 < C_{Z_\theta} < 0$$

(d) C_{Z_δ}: C_{Z_δ}, like C_{X_δ}, is generally small with relation to the forces caused by airplane motions. However, certain missile designs have utilized their wings for longitudinal control and, under certain circumstances, this factor could become appreciable. The extreme value of C_{Z_δ} would be reached in the case of an all-movable

wing, where C_{Z_δ} would be of the order of the lift-curve slope of the wings. Hence the recommended range of C_{Z_δ} is

$$-6.0 < C_{Z_\delta} < 0$$

(e) $C_{Z_{D_\alpha}}$, $C_{Z_{D_\delta}}$, C_{Z_q}: These derivatives are usually small and are generally neglected.

Pitching-moment derivatives (equation (6)).- (a) C_{m_u}: This derivative is largely a function of the thrust and its moment arm. A value of C_{m_u} of -0.038 has been estimated for missile no. 2 of table I.

Recommended range of C_{m_u} is

$$-0.5 < C_{m_u} < 0.5$$

(b) C_{m_α}: C_{m_α} is approximately equal to $C_{L_\alpha} \dfrac{dC_m}{dC_L}$. $\dfrac{dC_m}{dC_L}$ on present-day designs can be as large as 0.25 and may become positive (unstable). Typical values of C_{m_α} for several modern missiles range from -0.40 to -0.80.

Recommended range of C_{m_α} is

$$-4.0 < C_{m_\alpha} < 2.0$$

(c) $C_{m_{D_\alpha}}$: The derivative $C_{m_{D_\alpha}}$ arises because of the lag between the change in angle of attack at the wing and the corresponding downwash at the tail. Following are values of $C_{m_{D_\alpha}}$:

Missile no. (see table I)	$C_{m_{D_\alpha}}$
1	-6.0
2	-6.25
5	-5.62

A satisfactory range for $C_{m_{D_\alpha}}$ would be

$$-20.0 < C_{m_{D_\alpha}} < 0$$

(d) C_{m_q}: This derivative denotes the rotary damping in pitch. A summary of existing C_{m_q} data indicates that this parameter is rarely greater than -15.0 for conventional aircraft. For missiles with large horizontal tails with respect to their wing area, larger values of C_{m_q} could be expected. Hence, the recommended range of C_{m_q} is:

$$-50.0 < C_{m_q} < 0$$

(e) C_{m_δ}: This derivative is a function of the elevator effectiveness and its linkage to the automatic control. On normal type elevator controls a value of $C_{m_{\delta_e}}$ of the order of -1.0 is not uncommon. The extreme case would arise where an all-moving wing or tail is used for control. For this case,

$$C_{m_\delta} = C_{L_\delta}\frac{l}{c}$$

Assuming a tail length of 2-chord lengths the recommended range of C_{m_δ} is,

$$-12.0 < C_{m_\delta} < 0$$

(f) $C_{m_{D_\delta}}$: This derivative could become nearly as large as $C_{m_{D_\alpha}}$ on a tailless missile with all-movable wings. The recommended range is

$$-15.0 < C_{m_{D_\delta}} < 0$$

Miscellaneous information.- (a) $\frac{m}{\rho SV}$: This factor is a measure of the mass characteristics of the missile and is about $2.0 \longrightarrow 4.0$

for the average missile. Provision should be made to allow a
range of

$$0 < \frac{m}{\rho S V} < 50.0$$

to provide a safe margin for future designs which may operate at
very high altitudes.

(b) K_Y: This factor should be of the same order of magnitude
as K_Z. The recommended range is therefore

$$0 < K_Y < 5.0$$

(c) μ_c: $\mu_c = \frac{m}{\rho S c}$. Values of this parameter as high as 400
have been obtained. It is believed that values of μ_c up to 1000
should be considered.

If it is considered necessary to include the cross derivatives;
that is, those parameters normally associated with the longitudinal
equations which are affected by lateral displacements and vice-versa
$\left(C_{m_p}, \ C_{l_q}, \ C_{D_\beta}, \ \text{etc.}\right)$, these values can be easily determined and
substituted into the equations in the appropriate places. In order
to account properly for these cross derivatives, however, the right
members of equations (1) to (6) must be generalized by including the
terms depending upon combined longitudinal and lateral motion.
Including these terms is equivalent to eliminating assumption 5,
page 6. Two of the most important of the cross derivatives are C_{m_p},
which at a maximum would equal $-C_{n_p}$, and C_{D_β}, which would
equal C_{D_α} for a cruciform aircraft configuration. Other cross
derivatives can be estimated from similar considerations.

CONCLUDING REMARKS

The values of the parameters given here are only approximate
and cover a much wider range than most missiles and pilotless air-
craft will obtain. However, some of the parameters may increase to
a value of infinity under certain conditions. Under these conditions,
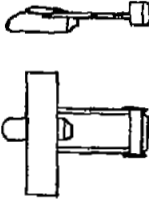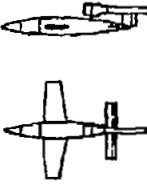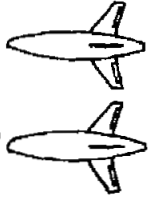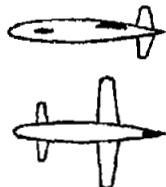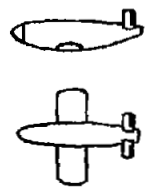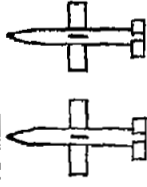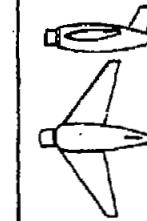particularly at very high altitudes in tenuous air, the concept of

dynamic airplane stability as it is seen today is altered somewhat, and the aerodynamic forces arising from the inherent stability of the aircraft become secondary to the mass forces and moments and to the moments and forces caused by control deflection.

Langley Memorial Aeronautical Laboratory
    National Advisory Committee for Aeronautics
        Langley Field, Va., May 22, 1947

TABLE I

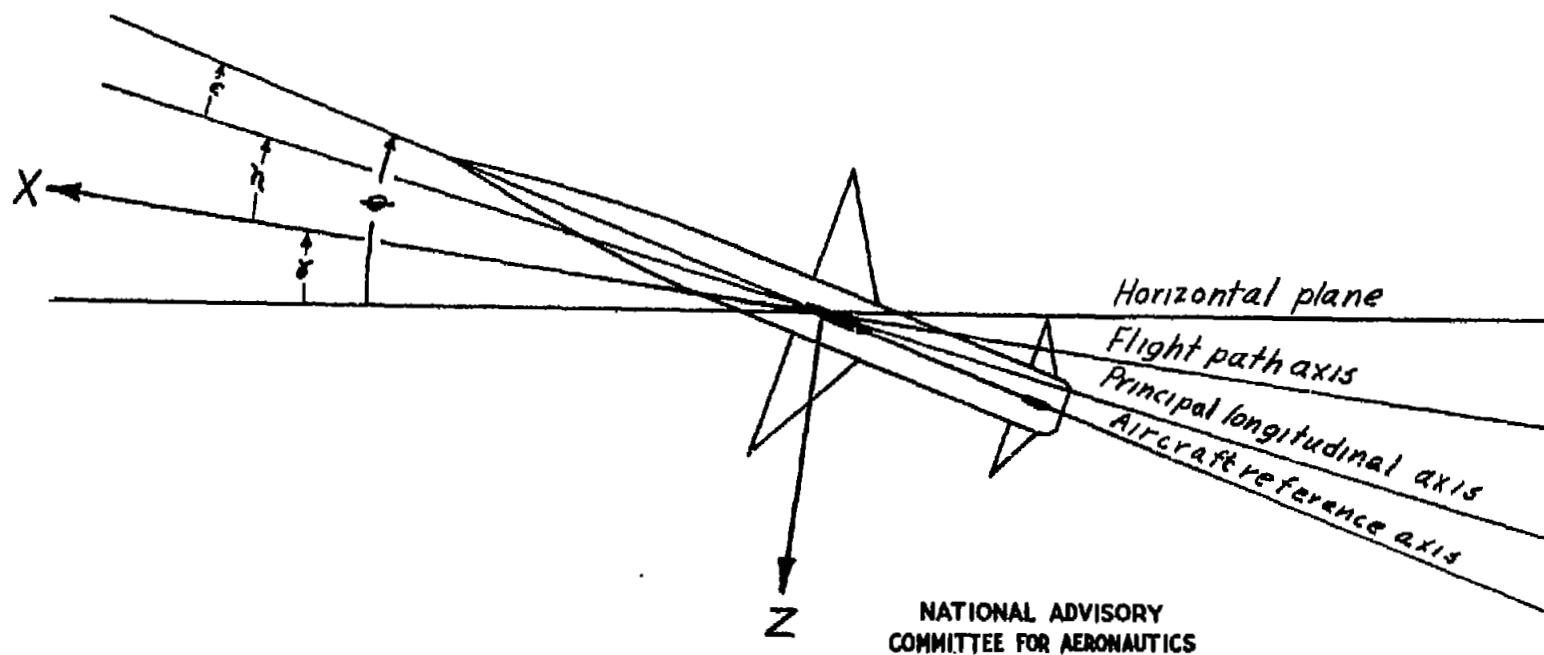Summary of Lateral Stability Derivatives for Various Missiles

Missile Number	1	2	3	4	5	6	7	8
Weight	2444	2642	600	980	1500	1500	883	
Wing Area (sq ft)	33.0	58.7	5.8	19.9	18.1	39.0	8.5	
Wing Span (ft)	12.0	17.0	5.0	11.0	8.5	14.2	5.5	
c_{Y_β}	−0.64	−0.35	−4.58	−0.63	−0.90	−0.40	−5.15	−1.36
c_{n_β}	0.183	0.085	0.22 to 0.44	0.126	0.036	0.075	0.044 to 0.146	0.03
c_{n_r}	−0.340	−0.204	−0.665	−0.337	−0.550	−0.245		−0.007
c_{n_p}	−0.026	−0.010		−.0042	−.0256	−.0022		−.0190
c_{l_β}	−0.069	−0.034		−0.109	−0.143	−0.060		−0.022
c_{l_p}	−0.43	−0.46		−0.42	−0.43	−0.37		−0.40
c_{l_r}	0.147	0.058		0.017	0.185	0.019		0.076

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Note :
   Longitudinal principal axis is axis of least inertia.

Horizontal plane
Flight path axis
Principal longitudinal axis
Aircraft reference axis

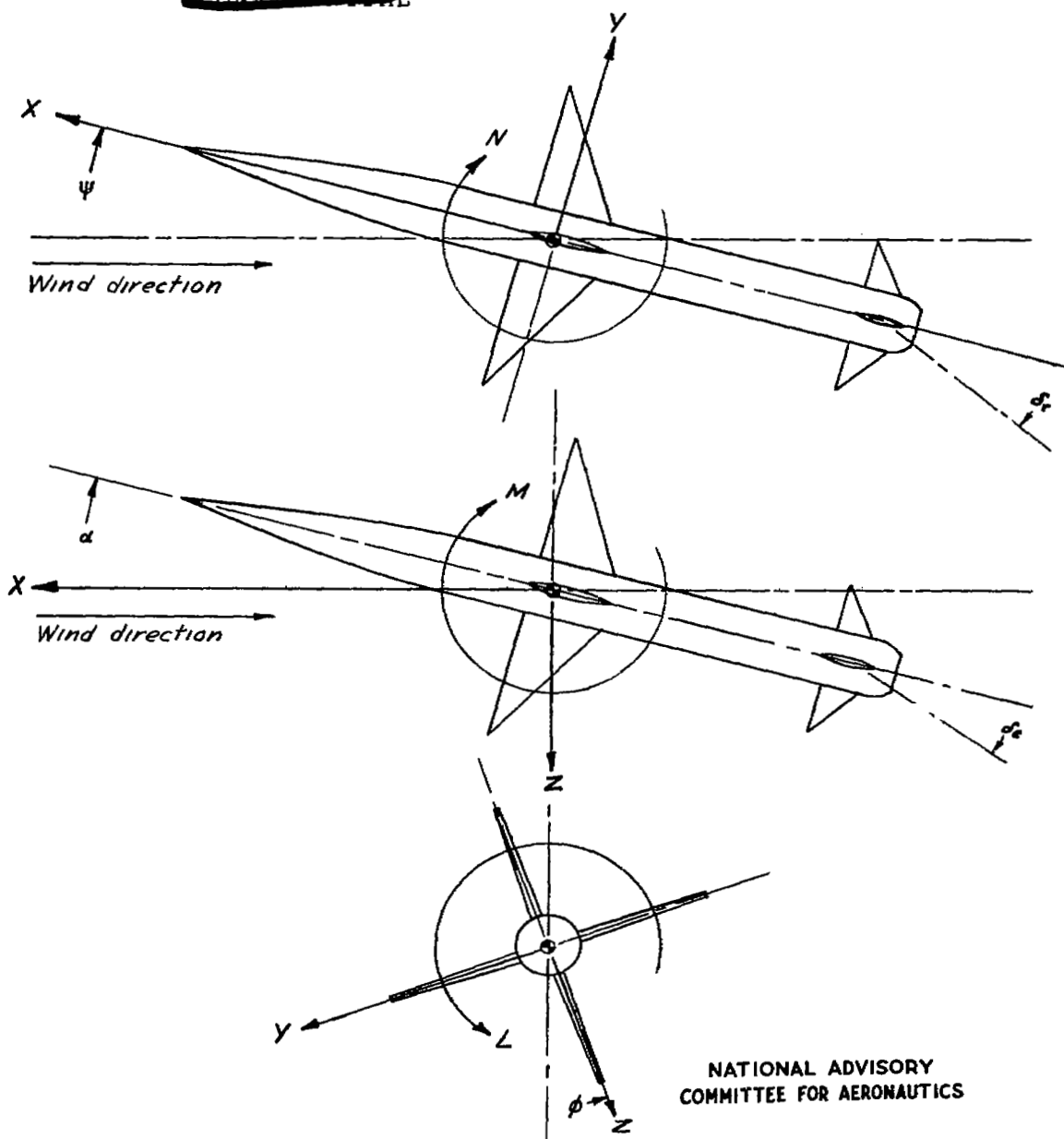**NATIONAL ADVISORY
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Figure 1 .— Longitudinal axes of aircraft

Figure 2.— System of stability axes. Arrows indicate
  positive directions of moments, forces, and control
  surface deflections.